THE NONSTEADY TEMPERATURE FIELD OF A SOLID UNDER THE NONSYMMETRIC AND TIME-VARIABLE CONDITIONS OF HEAT TRANSFER

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A method is proposed for the approximate solution of nonsteady heat-conduction problems under the nonsymmetric and time-variable boundary conditions of the third kind, applicable to bodies of arbitrary shape.

The solution presented in [1] for the problem of a nonsteady temperature field under the nonsymmetric conditions of heat transfer, on which [2] is based, makes it possible to calculate the temperature fields in a plate only under the condition that the temperatures of the media are constant.

It was demonstrated in [3] that for the solution of problems of nonsteady heat conduction under time-variable boundary conditions it is necessary and sufficient to find a number of solutions for the problems under constant boundary conditions.

These solutions should be treated as generalized functions or thermal characteristics of the body, whose use will make it possible to solve the problems under variable boundary conditions.

The proposed study is a further development of [3] and contains a discussion of the approximate numerical method for the solution of heat-conduction problems in bodies of both classical and arbitrary shape under nonsymmetric and time-variable conditions of heat transfer at the boundaries.

In examining problems with heat transfer at two contours, we note that the generalized temperature functions (thermal characteristics) for the characteristic points of the body are evidently functions of the form

$$\bar{\theta} = f(\mathrm{Bi_I},\ \mathrm{Bi_{II}},\ \mathrm{Fo}).$$

We will present $\bar{\theta}$ as the sum $\bar{\theta} = \bar{\theta}^*(\mathrm{Bi_I}, \mathrm{Fo}) + \Delta\bar{\theta}^*(\mathrm{Bi_I}, \mathrm{Bi_{II}}, \mathrm{Fo})$. Here $\bar{\theta}^*(\mathrm{Bi_I}, \mathrm{Fo})$ denotes the solutions derived for the case in which there is no heat transfer at the second contour $\mathrm{Bi_{II}} = 0$, and we will regard these as reference functions. Then $\Delta\bar{\theta}^*$ yields a correction factor for the effect of heat transfer at the second contour and will be a function of $\mathrm{Bi_{II}}$ with respect to the parameter $\mathrm{Bi_I}$. The numerical values of the function $\Delta\bar{\theta}^*$ for fixed values of Fo are defined as the difference between $\bar{\theta}$ and $\bar{\theta}^*$ for specified values of $\mathrm{Bi_I}$ and $\mathrm{Bi_{II}}$.

If the functions $\bar{\theta}^*$ are independent of the temperature t_{mI} of the medium and of t_i for the initial distribution, the functions $\Delta\bar{\theta}^*$ cannot treated as independent of the ratio $t_{mII} - t_i/t_{mI} - t_i = U_{mII}$, since in the general case $t_{mI}(\tau) \neq t_{mII}(\tau)$. If we define the function $\Delta\bar{\theta}^*$ for two fixed values of U_{mII}, e.g., $\Delta\bar{\theta}_a^*[\mathrm{Bi_I}, \mathrm{Bi_{II}}, \mathrm{Fo}, U_{mII}(a)]$ and $\Delta\bar{\theta}_b^*[\mathrm{Bi_I}, \mathrm{Bi_{II}}, \mathrm{Fo}, U_{mII}(b)]$, where $U_{mII}(a)$ is the value of U_{mII} for $t_{mII} = t_{mI}$, and $U_{mII}(b)$ when $t_{mII} = t_i$, under the conditions of linear interpolation in the interval from $U_{mII}(a)$ to $U_{mII}(b)$ the function $\Delta\bar{\theta}^*$ for any value of U_{mII} can be found as

$$\Delta\bar{\theta}^*[\mathrm{Bi_I},\ \mathrm{Bi_{II}},\ \mathrm{Fo},\ U_{mII}] = \frac{[U_{mII}(a) - U_{mII}]\,\Delta\bar{\theta}_b^* - [U_{mII}(b) - U_{mII}]\,\Delta\bar{\theta}_a^*}{U_{mII}(a) - U_{mII}(b)}.$$

As demonstrated by the results of the investigation, the linear interpolation yields the values of $\Delta\bar{\theta}^*$ which satisfy the requirements of accuracy for engineering calculations.

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TABLE 1. The Functions $\Delta\ \bar{\theta}^* = f\,[Bi_I,\ Bi_{II},\ Fo]$ for the Point at a Distance $x/\delta = 0.5$ from the Edge of the Plate when $U_{mII} = 100\%$ and $U_{mII} = 0\%$

Fo	$Bi_I=1;\ Bi_{II}=0,25$		$Bi_I=1;\ Bi_{II}=1$		$Bi_I=1;\ Bi_{II}=2$		$Bi_I=1;\ Bi_{II}=4$		$Bi_I=1;\ Bi_{II}=8$	
	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$
0,016	0	0	0	0	0	0	0	0	0	0
0,04	0	0	0,003	—0,002	0,01	—0,002	0,028	—0,002	0,028	—0,002
0,08	0,002	—0,004	0,026	—0,005	0,051	—0,005	0,101	—0,004	0,126	—0,004
0,159	0,022	—0,005	0,082	—0,01	0,160	—0,010	0,245	—0,010	0,280	—0,010
0,240	0,035	—0,008	0,133	—0,014	0,229	—0,014	0,325	—0,014	0,385	—0,015
0,319	0,056	—0,010	0,178	—0,020	0,290	—0,020	0,388	—0,023	0,448	—0,031
0,400	0,072	—0,015	0,210	—0,024	0,331	—0,025	0,430	—0,034	0,486	—0,048
0,557	0,087	—0,018	0,246	—0,053	0,375	—0,061	0,460	—0,080	0,504	—0,100
0,795	0,102	—0,037	0,274	—0,099	0,385	—0,125	0,454	—0,160	0,474	—0,194

Fo	$Bi_I=2;\ Bi_{II}=0,25$		$Bi_I=2;\ Bi_{II}=1$		$Bi_I=2;\ Bi_{II}=2$		$Bi_I=2;\ Bi_{II}=4$		$Bi_I=2;\ Bi_{II}=8$	
	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$
0,016	0,0	0	0	0	0	0	0	0	0	0
0,04	0,0	0	0,005	0	0,010	0	0,02	0	0,032	0
0,08	0,008	0	0,032	—0,002	0,061	—0,004	0,097	—0,002	0,137	—0,002
0,159	0,028	0	0,097	—0,003	0,155	—0,008	0,230	—0,003	0,295	—0,006
0,240	0,045	—0,002	0,146	—0,004	0,223	—0,014	0,319	—0,008	0,386	—0,015
0,319	0,059	—0,003	0,180	—0,012	0,264	—0,034	0,366	—0,033	0,428	—0,042
0,400	0,073	—0,005	0,208	—0,025	0,294	—0,047	0,393	—0,053	0,445	—0,067
0,557	0,087	—0,014	0,237	—0,049	0,310	—0,096	0,397	—0,110	0,432	—0,136
0,795	0,102	—0,045	0,220	—0,120	0,270	—0,189	0,332	—0,229	0,338	—0,264

Fo	$Bi_I=4;\ Bi_{II}=0,25$		$Bi_I=4;\ Bi_{II}=1$		$Bi_I=4;\ Bi_{II}=2$		$Bi_I=4;\ Bi_{II}=4$		$Bi_I=4;\ Bi_{II}=16$	
	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$	$\Delta\bar{\theta}^*_a(100)$	$\Delta\bar{\theta}^*_b(0)$
0,016	0	0	0	0	0	0	0	0	0	0
0,04	0,005	0	0,010	0	0,020	0	0,030	0	0,05	0
0,08	0,018	0	0,041	0	0,075	0	0,094	0	0,175	0
0,159	0,025	0	0,091	—0,003	0,169	—0,004	0,214	—0,004	0,335	—0,004
0,240	0,043	—0,002	0,136	—0,007	0,239	—0,014	0,292	—0,017	0,408	—0,022
0,319	0,049	—0,005	0,160	—0,025	0,270	—0,041	0,323	—0,049	0,425	—0,065
0,400	0,060	—0,012	0,180	—0,035	0,288	—0,063	0,335	—0,075	0,415	—0,105
0,557	0,059	—0,032	0,175	—0,079	0,267	—0,133	0,303	—0,155	0,355	—0,195
0,795	0,053	—0,059	0,150	—0,148	0,205	—0,225	0,222	—0,265	0,247	—0,317

It is obvious that for a flat plate the functions $\Delta\bar{\theta}^*$ can be derived if from the solution of [2] for fixed values of Bi_I and Bi_{II} we subtract the corresponding solutions for the plate [4] for the same values of Bi_I.

To derive the functions $\bar{\theta}^*$ and $\Delta\bar{\theta}^*$, we used an analog computer.

In determining the thermal characteristics (the functions $\bar{\theta}^*$ and $\Delta\bar{\theta}^*$) at the characteristic point on bodies of complex configuration with heat transfer at two contours, the experiments on the analog computer may be formulated so that it becomes possible to find both $\bar{\theta}^* = f(Bi_I,\ Fo)$, and $\Delta\bar{\theta}^* = f(Bi_{II},\ Fo)$ with respect to the parameter Bi_I.

The numerical values of $\bar{\theta}^*$ for Bi_I = const in the case of a series of values for Fo are determined from the results of the electrical modeling

$$\bar{\theta}^*(Bi_I,\ Fo) = \frac{U_1(Fo) - U_i}{U_{mI} - U_H}.$$

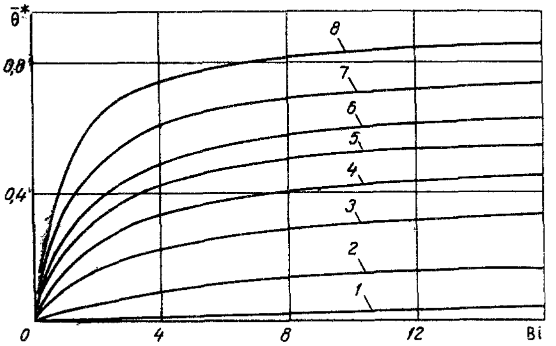


Fig. 1. The functions $\bar{\theta}^* = f(Bi_I)$ with respect to the parameter Fo for a point separated from the edge of the plate through a distance x/δ = 0.5: 1) Fo = 0.04; 2) 0.08; 3) 0.159; 4) 0.240; 5) 0.319; 6) 0.400; 7) 0.557; 8) 0.795.

The numerical values of the functions $\Delta\bar{\theta}^*$ when Bi_{II} = const and for fixed values of Bi_I and $U_{mII}(a)$ for a number of Fo values are found as

$$\Delta\bar{\theta}_a^* = \frac{U_{2a}(\text{Fo}) - U_i(\text{Fo})}{U_{mI} - U_i}.$$

Analogously we determine the function $\Delta\bar{\theta}^*$ for $U_{mII}(b)$

$$\Delta\bar{\theta}_b^* = \frac{U_{2b}(\text{Fo}) - U_i(\text{Fo})}{U_{mI} - U_i}.$$

With the functions $\Delta\bar{\theta}_a^*$ and $\Delta\bar{\theta}_b^*$ we find $\Delta\bar{\theta}^* = f(Bi_I, Bi_{II}, \text{Fo}, U_{mII})$ for any U_{mII}.

The range of the investigations on the analog computer must be such that all of the values of the functions $\bar{\theta} = f(Bi_I, Bi_{II}, \text{Fo})$ can be determined by interpolation of the functions $\bar{\theta}^*$ and $\Delta\bar{\theta}^*$ in the intervals of variation in Bi_I and Bi_{II}.

We will demonstrate through a series of examples that if for the characteristic points we have found the function $\bar{\theta}^*$, through application of the method described in [3] it is possible to solve the problem of a nonsteady temperature field under the nonsymmetric conditions of heat transfer.

Let us examine an unbounded plate. As was noted earlier, the functions $\bar{\theta}^*$ and $\Delta\bar{\theta}^*$ for the plate can be found from the solutions given in [2] and [4]. At the same time, they can be found by the method of electrical modeling for a body of arbitrary configuration as well.

We note that if we find the numerical values of $\bar{\theta}^* = f(Bi_I, \text{Fo})$ as a result of electrical modeling for fixed values of Bi_I, we can rationally construct a family of the functions $\bar{\theta}^* = f(Bi_I)$ with respect to the Fo number (see Fig. 1), since in this case for any Bi_I we can find, from the graph, the numerical values of $\bar{\theta}^*$ for fixed values of Fo.

For the point situated at a distance x/δ from the edge of the plate, in Table 1 we present the functions $\Delta\bar{\theta}^*$ derived from processing the electrical-modeling data relating to problems of heating with nonsymmetric but time-constant boundary conditions of heat transfer for the cases in which $U_{mII}(a)$ = 100% and $U_{mII}(b)$ = 0%.

We will test the numerical method proposed in [3] for the calculation of the nonsteady temperature field in a solid under nonsymmetric conditions of heat transfer at the boundary on the example of a problem whose solution is given in [1]. Having replaced the continuous functions $Bi_I = 4 - 3\exp(-2\,\text{Fo})$ and $Bi_{II} = 2 - 1.5\exp(-\text{Fo})$ with piecewise-step functions, for fixed values of Bi_I and Bi_{II} we will determine the function $\bar{\theta}$ (see Table 2). Using the method from [3], we find the change in the relative temperature at the point x/δ = 0.5 and we compare with the results from [1] (see Table 3a). We will demonstrate that the recommended method of numerical solution for problems of nonsteady heat conduction is suitable not only for the case of the time variables $Bi_I = f_1(\tau)$ and $Bi_{II} = f_2(\tau)$, but also for the variable temperatures of the media.

We will solve numerically and compare with the results derived from the analog modeling of the problem of nonsymmetric heating of a plate for the case in which the coefficients of heat transfer at the boundaries remain constant and it is only the temperatures of the media that vary with time. Comparison of the solutions is given for the point x/δ = 0.5. The conditions at the boundaries are formulated as follows: $Bi_I = 4$; $Bi_{II} = 16$;

$$t_{mI} = 420 + 285\cdot\text{Fo} \quad \text{when} \quad 0 \leqslant \text{Fo} \leqslant 0.24,$$
$$t_{mI} = 454.5 + 146.5\cdot\text{Fo} \quad \text{when} \quad 0.24 \leqslant \text{Fo} \leqslant 0.8,$$
$$t_{mII} = 330 + 71.2\cdot\text{Fo} \quad \text{when} \quad 0 \leqslant \text{Fo} \leqslant 0.24,$$
$$t_{mII} = 339 + 36.7\cdot\text{Fo} \quad \text{when} \quad 0.24 \leqslant \text{Fo} \leqslant 0.8.$$

The initial distribution is uniform, i.e., t_i = 300°K.

The laws governing the change in the temperatures $t_{mI} = f(\tau)$ and $t_{mII} = f(\tau)$ were chosen so that the ratio $t_{mII} - t_i/t_{mI} - t_i$ remains constant throughout the entire time during which the problem is being solved.

TABLE 2. The Functions $\theta = f[Bi_I, Bi_{II}, Fo]$ for Point at a Distance $x/\delta = 0.5$ from the Edge of the Plate, when $U_{mII} = 75\%$

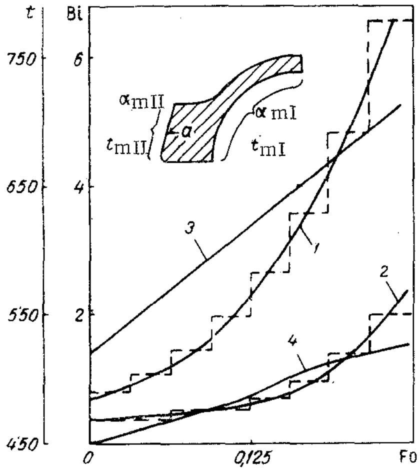
Fo	Bi_I=1,17; Bi_{II}=0,58			Bi_I=1,5; Bi_{II}=0,58			Bi_I=1,74; Bi_{II}=0,735			Bi_I=1,92; Bi_{II}=0,735			Bi_I=2,17; Bi_{II}=0,852			Bi_I=2,5 Bi_{II}=0,952		
	$\bar{\theta}^*$	$\Delta\bar{\theta}^*$	$\bar{\theta}$	$\bar{\theta}^*$	$\Delta\bar{\theta}^*$	$\bar{\theta}$	$\bar{\theta}^*$	$\Delta\bar{\theta}^*$	$\bar{\theta}$	$\bar{\theta}^*$	$\Delta\bar{\theta}^*$	$\bar{\theta}$	$\bar{\theta}^*$	$\Delta\bar{\theta}^*$	$\bar{\theta}$	$\bar{\theta}^*$	$\Delta\bar{\theta}^*$	$\bar{\theta}$
0,04	0,005	0,001	0,006	0,007	0,001	0,008	0,010	0,002	0,012	0,011	0,002	0,013	0,012	0,003	0,015	0,013	0,004	0,017
0,08	0,037	0,009	0,046	0,046	0,011	0,057	0,050	0,016	0,066	0,055	0,017	0,072	0,058	0,02	0,078	0,065	0,024	0,089
0,16	0,112	0,031	0,143	0,130	0,036	0,166	0,145	0,05	0,195	0,155	0,053	0,208	0,165	0,061	0,226	0,180	0,068	0,248
0,24	0,172	0,058	0,23	0,205	0,062	0,267	0,222	0,08	0,302	0,236	0,081	0,317	0,250	0,092	0,342	0,270	0,102	0,372
0,32	0,227	0,08	0,307	0,266	0,081	0,347	0,292	0,095	0,387	0,310	0,094	0,404	0,325	0,113	0,438	0,345	0,122	0,467
0,40	0,282	0,096	0,378	0,325	0,096	0,421	0,355	0,116	0,471	0,372	0,116	0,488	0,390	0,120	0,510	0,417	0,138	0,555
0,56	0,370	0,11	0,48	0,420	0,109	0,529	0,450	0,130	0,580	0,470	0,129	0,599	0,495	0,141	0,636	0,522	0,146	0,668
0,80	0,487	0,116	0,603	0,550	0,114	0,664	0,590	0,134	0,724	0,617	0,133	0,750	0,642	0,141	0,783	0,672	0,139	0,811

Fo	Bi_I=2,79; Bi_{II}=1,05			Bi_I=3,01; Bi_{II}=1,13			Bi_I=3,19; Bi_{II}=1,25			Bi_I=3,43; Bi_{II}=1,33			Bi_I=3,43; Bi_{II}=1,4			Bi_I=3,43; Bi_{II}=1,43		
	$\bar{\theta}^*$	$\Delta\bar{\theta}^*$	$\bar{\theta}$	$\bar{\theta}^*$	$\Delta\bar{\theta}^*$	$\bar{\theta}$	$\bar{\theta}^*$	$\Delta\bar{\theta}^*$	$\bar{\theta}$	$\bar{\theta}^*$	$\Delta\bar{\theta}^*$	$\bar{\theta}$	$\bar{\theta}^*$	$\Delta\bar{\theta}^*$	$\bar{\theta}$	$\bar{\theta}^*$	$\Delta\bar{\theta}^*$	$\bar{\theta}$
0,04	0,014	0,006	0,02	0,015	0,006	0,021	0,016	0,007	0,023	0,017	0,008	0,025	0,017	0,009	0,026	0,017	0,009	0,026
0,08	0,07	0,028	0,098	0,072	0,03	0,102	0,075	0,034	0,109	0,077	0,037	0,114	0,077	0,038	0,115	0,077	0,039	0,116
0,16	0,190	0,073	0,263	0,197	0,076	0,273	0,205	0,082	0,287	0,207	0,087	0,294	0,207	0,09	0,297	0,207	0,092	0,299
0,24	0,282	0,109	0,391	0,290	0,113	0,403	0,300	0,120	0,420	0,307	0,126	0,433	0,307	0,130	0,437	0,307	0,133	0,440
0,32	0,362	0,129	0,491	0,373	0,132	0,505	0,382	0,133	0,515	0,390	0,143	0,533	0,390	0,148	0,538	0,390	0,150	0,540
0,40	0,435	0,144	0,579	0,445	0,147	0,592	0,455	0,153	0,608	0,462	0,156	0,618	0,462	0,161	0,623	0,462	0,164	0,626
0,56	0,540	0,147	0,687	0,552	0,145	0,697	0,565	0,146	0,711	0,576	0,144	0,720	0,576	0,148	0,724	0,576	0,149	0,725
0,80	0,690	0,133	0,823	0,705	0,122	0,827	0,715	0,116	0,831	0,722	0,106	0,828	0,722	0,107	0,829	0,722	0,108	0,830

TABLE 3. Comparison of Variations in Temperature at a Point at a Distance $x/\delta = 0.05$ from the Edge of the Plate in Problems with Nonsymmetric Conditions of Heat Transfer

	Fo	0,05	0,1	0,15	0,20	0,25	0,30	0,35	0,40	0,50	0,60	0,70	0,80	0,90	1,0
a	t_{mI}						t_{cI}=const								
	$t_{mI} - t_i$						$0{,}8t_{mI}$								
	$\frac{t_{mII} - t_i}{t_{mI} - t_i}$						0,75								
	Bi_I	1,17	1,5	1,74	1,92	2,17	2,17	2,50	2,50	2,79	3,01	3,19	3,43	3,43	3,43
	Bi_{II}	0,58	0,58	0,735	0,735	0,852	0,852	0,952	0,952	1,05	1,13	1,25	1,33	1,40	1,43
	$\bar{\theta}$	0,01	0,07	0,147	0,222	0,292	0,355	0,417	0,472	0,575	0,652	0,705	0,750	0,790	0,825
	T_p/t_{mI}	0,208	0,256	0,318	0,377	0,433	0,484	0,534	0,577	0,658	0,720	0,765	0,800	0,831	0,860
	T/t_{mI}*)	—	0,258	—	0,369	—	0,470	--	0,572	0,653	0,711	0,771	0,807	0,838	0,861
b	Fo	0,02	0,04	0,06	0,08	0,10	0,12	0,14	0,16	0,18	0,20	0,22	0,24	0,26	0,28
	t_{mIi}, °K	423	429	435	441	447	453	459	465	471	476	481	486	489	492
	$t_{mIi} - t_i$	123	129	135	141	147	153	159	165	171	176	181	186	189	192
	$\bar{\theta}_i$	0	0,0095	0,0305	0,0765	0,115	0,153	0,192	0,222	0,254	0,279	0,290	0,310	0,327	0,342
	$\bar{\theta}_{cir}$	0,01	0,032	0,08	0,120	0,160	0,200	0,232	0,265	0,287	0,302	0,322	0,335	0,351	0,367
	T_{cal} °K	301	304	311	317	323	330	337	344	349	353	358	362	366	371
	T_s, °K	—	303	—	315	—	—	—	342	—	—	—	365	—	—
	Fo	0,30	0,32	0,34	0,36	0,38	0,40	0,48	0,56	0,64	0,72	0,80			
	t_{mIi}, °K	495	498	501	504	507	511	517	530	544	559	574			
	$t_{mIi} - t_i$	195	198	201	204	207	211	217	230	244	259	274			
	$\frac{t_{mIi} - t_i}{t_{mIi+1} - t_i}$	0,987	0,987	0,987	0,987	0,987	0,987	0,972	0,942	0,942	0,942	0,942			
	$\bar{\theta}_i$	0,358	0,376	0,394	0,405	0,412	0,417	0,422	0,452	0,475	0,487	0,495			
	$\bar{\theta}_{cir}$	0,385	0,402	0,415	0,422	0,427	0,435	0,480	0,500	0,515	0,522	0,530			
	T_{cal} °K	375	380	383	386	389	392	404	415	425	435	445			
	T_s, °K	—	382	—	—	—	396	—	414	—	—	439			

*The values of T/T_{mI} taken for [1] have been calculated in accordance with Vanichev.



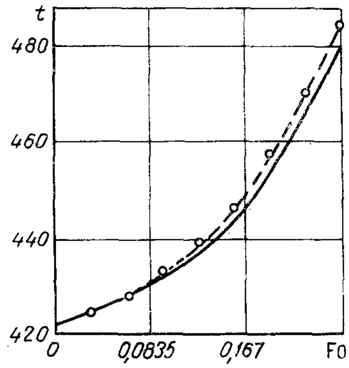


Fig. 3

Fig. 2. Specification of the boundary conditions of heat transfer on the model of a lateral cross section of the housing: 1) change in heat-transfer intensity at the inside contour, Bi_I; 2) the same, at the outside contour, Bi_{II}; 3) change in the temperature of the medium (°K) at the inside contour ($t_{mI\,\bar{\theta}}$); 4) the same, at the outside contour (t_{mII}).

Fig. 3. Comparison of the change in temperature at point *a* in Fig. 2 in the heating regime, as derived by calculation and electrical modeling (the curve denotes calculation; the points denote the values of the temperature at individual instants of time, found through modeling on a USM-1 analog computer).

In this case there was a substantial simplification in the definition of the function $\Delta\bar{\theta}^* = f[Bi_I, Bi_{II}, Fo, U_{mII}]$, since U_{mII} = const. Using the data of Fig. 1 and Table 1, let us determine the numerical values of the functions $\bar{\theta}$ with $Bi_I = 4$ and $Bi_{II} = 16$ for a number of fixed values of Fo.

We will replace the continuous function $t_{mI} = f(\tau)$ with a piecewise-step function with constancy intervals equal to Fo = 0.02 at the beginning and Fo = 0.08 at the end of the investigated heating regime.

The results from the comparison of the solutions are shown in Table 3. It is obvious that if for the characteristic point of a body of complex configuration with heat transfer at two contours, e.g., the lateral cross section of the casing of a steam turbine (see Fig. 2) if the analog computer as a result of electrical modeling yields the functions $\bar{\theta}^*$ and $\Delta\bar{\theta}^*$, without resorting to the modeling, it is possible to solve a problem relating to a nonsteady temperature field for some arbitrary time variation in the conditions of heat transfer at the boundaries. To alter the boundary conditions in accordance with Fig. 2, we compared the solutions obtained on the USM-1 with the numerical results found from the above-described methods (see Fig. 3). The comparison of the numerical results through the utilization of the method described in [3] and the solutions derived by analytical methods for bodies of classical shape, as well as those found on the analog computer for bodies of complex configuration under time-variable boundary conditions of heat transfer thus makes it possible to state that the proposed method of solving problems of nonsteady heat conduction expands the potential for the more complete and integrated utilization of analog and digital computers in power engineering, separating the problems so that the analog computer is used exclusively for preparation of information for the digital computer.

NOTATION

$t_{mI}(\tau)$ is the temperature of the medium at the first contour;
$t_{mII}(\tau)$ is the temperature of the medium at the second contour;
δ is the thickness of the plate;
x is the distance from the first contour to the point under consideration;
$U_1(Fo)$ is the potential measured at the point in modeling the problem of heat transfer at only the first contour on an analog computer;

$U_{2a}(Fo)$ is the potential measured at that same point, with modeling of the problem with heat transfer at two contours, when $t_{mII} = t_{mI}$;

$U_{2b}(Fo)$ is the potential measured at that same point, in modeling the problem with heat transfer at two contours, when $t_{mII} = t_i$;

$T_{cal} = \Theta_{cir}(t_{mI} - t_i) + t_i$ denotes the temperature of the body at the point, as derived numerically;

T_s is the temperature of the body at the point, derived by simulation (modeling).

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